

## PHYSICS - I LABORATORY

MANUAL FOR ENGINEERING UNDERGRADUATES

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## 0. ERROR ANALYSIS

### 0.1. Error

All experimentally determined physical quantities contain some degree of error or uncertainty. The error in a measurement is defined as the difference between the measured value and the real value of a physical quantity determined in the experiment.

The numerical value of a measured physical quantity cannot be expressed with its real value due to the experimental errors. The sensitivity and the confidentiality of a measurement are determined with the level of errors, which occur in experiments. An experimental result does not have any scientific meaning unless the error level is provided. Thus, the error, which arises in measurements of the experimental studies, should be determined at each step. There are two kinds of error called systematical and statistical:


### 0.2. Absolute Error

The difference between the real value $X_{0}$ and the measured value $X$ of a physical quantity is called the absolute error in $X_{0}$.

$$
\begin{gathered}
\text { Absolute Error }= \pm \Delta X=X_{0}-X \\
\text { RealValue }=X_{0}=X \pm \Delta X
\end{gathered}
$$

Since the real value $X_{0}$ is not exactly known, the value of $\Delta X$ cannot be known exactly, but it can be approximately calculated with some methods. The level of absolute error can be determined with the difference between the measured value and the best value that is obtained in all measurements.

### 0.3. Relative Error

The relative error is defined as the ratio of the absolute error, $\Delta X$, to the measured value, $X$. Multiplying the relative error by 100 gives the percentage error:

$$
\begin{gathered}
\text { Relative Error }=\frac{\Delta X}{X} \\
\text { PercentageError }=\frac{\Delta X}{X} \times 100
\end{gathered}
$$

### 0.4. Arithmetical Mean

The values obtained by repeating the same measurements many times form a distribution around a certain value. Such a central value is called "mean".

If $X_{1}, X_{2}, X_{3}, \ldots, X_{\mathrm{N}}$ are the values obtained in repeating measurements, the arithmetical mean of them

$$
\bar{X}=\frac{X_{1}+X_{2}+X_{3}+\cdots+X_{N}}{N}=\frac{\sum_{i=1}^{N} X_{i}}{N}
$$

$\bar{X}$ is the value of $X$ in the best approximation. Then, if a quantity is measured $N$ times, the mean value can be considered the result of measurements.

### 0.5. Standart Error in the Mean Value

The mean value can be different depending on the distribution of values obtained in a set of measurements. Therefore, the mean value has also a standard deviation and this is called "standard error", which is calculated by

$$
\Delta X=\sqrt{\frac{\sum_{i=1}^{N}\left(X_{i}-\bar{X}\right)^{2}}{N(N-1)}}
$$

### 0.6. The Error in a Single Measurement with a Tool

In the cases in which it is not possible to repeat measurements, the most appropriate way to determine the error is to take the half of the least amount that can be measured with the tool used in the measurement. For instance, the largest error for a ruler that can measure the least length of 1 mm should be taken as $\Delta X=0.5 \mathrm{~mm}$.

### 0.7. Error in Composite Quantities

If a result depends on various quantities and the error in each of them is known, the total error in the results should be calculated. The standard error in $Q$ is generally calculated as follows:

$$
\Delta Q=\sqrt{\left(\frac{\partial Q}{\partial x} \Delta x\right)^{2}+\left(\frac{\partial Q}{\partial y} \Delta y\right)^{2}+\left(\frac{\partial Q}{\partial z} \Delta z\right)^{2}+\cdots}
$$

where $\frac{\partial Q}{\partial x}$ is the partial derivation of $Q$ with respect to $x, \frac{\partial Q}{\partial y}$ is the partial derivation of $Q$ with respect to $y$ and $\frac{\partial Q}{\partial z}$ is the partial derivation of $Q$ with respect to $z$.

### 0.8. Error Calculation in Fundamental Algebra

- In Addition and Subtraction:

$$
Q=x+y \text { or } Q=x-y \quad \Rightarrow \quad \Delta Q=\Delta x+\Delta y
$$

- In Multiplication and Division:

$$
Q=x \cdot y \text { or } Q=\frac{x}{y} \quad \Rightarrow \quad \frac{\Delta Q}{Q}=\frac{\Delta x}{x}+\frac{\Delta y}{y}
$$

- In Exponential Functions:

$$
Q=x^{n} \quad \Rightarrow \quad \frac{\Delta Q}{Q}=n \frac{\Delta x}{x}
$$

- In Trigonometric Functions:
$Q=\sin x$, if the error in $x$ is $\Delta x$ and $x, \Delta x$ are given in radian

$$
\Delta Q=\cos x \cdot \Delta x \cdot \frac{\pi}{180}
$$

### 0.9. Calculating Error of a Quantity measured over a Graphic



The slope of the line drawn in the graphic can be found by

$$
R=\tan \theta=\frac{V_{2}-V_{1}}{I_{2}-I_{1}}
$$

Following the error calculation in multiplication and division, the error in $R$ can be written as

$$
\frac{\Delta R}{R}=\frac{\Delta\left(V_{2}-V_{1}\right)}{V_{2}-V_{1}}+\frac{\Delta\left(I_{2}-I_{1}\right)}{I_{2}-I_{1}}
$$

where $\Delta\left(V_{2}-V_{1}\right)=\Delta V_{2}+\Delta V_{1}$ denotes the error in $\left(V_{2}-V_{1}\right)$ and $\Delta\left(I_{2}-I_{1}\right)=\Delta I_{2}+\Delta I_{1}$ denotes the error in $\left(I_{2}-I_{1}\right)$. Then the relative error in $R$ is obtained by

$$
\frac{\Delta R}{R}=\frac{\Delta V_{2}+\Delta V_{1}}{V_{2}-V_{1}}+\frac{\Delta I_{2}+\Delta I_{1}}{I_{2}-I_{1}}
$$

Determining over the graphic
$\Delta V=\Delta V_{2}=\Delta V_{1}$ : The measure between two tics next to each other on $V$-axis
$\Delta I=\Delta I_{2}=\Delta I_{1}$ : The measure between two tics next to each other on $I$-axis
Substituting yields

$$
\frac{\Delta R}{R}=\frac{2 \Delta V}{V_{2}-V_{1}}+\frac{2 \Delta I}{I_{2}-I_{1}}
$$

## 1. MOTION IN ONE DIMENSION

## Equipment

Purpose : The main purpose of this experiment is to study and analyze:

- The position and velocity of the motion with constant velocity,
- The acceleration of a straight-line motion with constant acceleration,
- Horizontal projectile (two-dimensional) motion of an object moving on an inclined air table,
- Conservation of linear momentum.


### 1.1. Experimental Principle

### 1.1.1. Straight Line Motion with Constant Velocity

When a particle moves along a straight line, we can describe its position with respect to an origin (O), by means of a coordinate (such as $x$ ). If there is no net force acting on a moving object, it moves on a straight line with a constant velocity. The particle's average velocity ( $v_{a v}$ ) during a time interval ( $\Delta t=t_{2}-t_{1}$ ) is equal to its displacement ( $\Delta x=x_{2}-x_{1}$ ) divided by $\Delta t$ :

$$
\begin{equation*}
v_{a v}=\frac{x_{2}-x_{1}}{t_{2}-t_{1}}=\frac{\Delta x}{\Delta t} \tag{1}
\end{equation*}
$$

From the Eq.1, the average velocity is the displacement $(\Delta x)$ divided by the time interval ( $\Delta t$ ) during which the displacement occurs. If we plot a graph $x$ versus $t$, then we will have a straight line with a slope. The slope of the line gives the average velocity of the motion.

For a displacement along the $x$-axis, the average velocity $\left(v_{a v}\right)$ of the object is equal to the slope of a line connecting the corresponding points on the graph of position versus time ( $x-t$ graph). The average velocity depends only on the total displacement $(x)$ that occurs during the motion time $(t)$. The position, $x(t)$ of an object moving in a straight line with constant velocity is given as a function of time as:

$$
\begin{equation*}
x(t)=x_{0}+v t \tag{2}
\end{equation*}
$$

If the object is at the origin with the initial position $x_{0}=0$, the equation of the motion becomes at any time:

$$
\begin{equation*}
x(t)=v t \tag{3}
\end{equation*}
$$

So, the object travels equal distance in the equal time intervals along a straight line (Fig.1.1).


Figure 1.1. Position as a function of time.

### 1.2. Straight Line Motion with a Constant Acceleration

When a particle slides straight down a frictionless inclined plane, its acceleration is constant, and it will move in a straight line down the plane. The magnitude of the acceleration depends on the angle at which the plane is inclined. If the inclination angle is $90^{\circ}$, the object will slide down with an acceleration which is equal to the Earth's gravitational acceleration $g$ with the magnitude of $9.8 \mathrm{~m} / \mathrm{s}^{2}$.

In this experiment, we will observe the motion of a puck moving in a straight line with a velocity changing uniformly. The back side of the air table is raised to form an inclined plane on the air table. The air table is inclined at an angle of $\theta$ with the horizontal plane as shown in the Fig.1.2.


Figure 1.2. The straight down motion on an inclined air table.
If you put the puck at the top of the inclined air table and let it slide down the plane, it will move downwards on a straight line but with increasing velocity. The rate of change of the velocity is the acceleration of the puck. If at time $t_{1}$, the puck is at the point $x_{1}$ with a velocity of $v_{1}$, then at a later time $t_{2}$, it will be at a point of $x_{2}$ with a velocity $v_{2}$.

The average acceleration of the puck in this time interval $\Delta t=t_{2}-t_{1}$ is defined as:

$$
\begin{equation*}
a_{a v}=\frac{\Delta v}{\Delta t}=\frac{v_{2}-v_{1}}{t_{2}-t_{1}} \tag{4}
\end{equation*}
$$

Suppose that at an initial time $t_{1}=0$, the puck is at the position of $x_{0}$ and has a velocity of $v_{1}=v_{0}$. At a later time $t_{2}=t$, it is in position $x$ and has a velocity of $v_{2}=v$. Then, the average acceleration will be equal to:

$$
\begin{equation*}
a_{a v}=\frac{v-v_{0}}{t-0} \tag{5}
\end{equation*}
$$

Then, the velocity of the puck will be:

$$
\begin{equation*}
v=v_{0}+a t \tag{6}
\end{equation*}
$$

If we consider the instantaneous acceleration (simply the acceleration) of the motion in the $x$-direction, it would be:

$$
\begin{equation*}
a=\lim _{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}=\frac{d v}{d t} \tag{7}
\end{equation*}
$$

The instantaneous acceleration in a straight line motion equals the instantaneous rate of change of velocity with time.

The equitation for an object's motion with constant acceleration in one dimension $(x)$ is:

$$
\begin{equation*}
x=x_{0}+v_{0} t+\frac{1}{2} a t^{2} \tag{8}
\end{equation*}
$$

where:
$x_{0}$ : The displacement at time $\mathrm{t}=0$ (initial displacement),
$v_{0}$ : The speed at time $\mathrm{t}=0$ (initial speed) and,
$a$ : The object's acceleration.

If the motion of object starts from rest $\left(x_{0}=0, v_{0}=0\right)$ at $t=0$, the object's position at any time $t$ (insta1ntaneous position) will be:

$$
\begin{equation*}
x=\frac{1}{2} a t^{2} \tag{9}
\end{equation*}
$$

A graph of Eq.9, that is, a $x-t$ graph for motion with constant acceleration, is always a parabola that passes through the origin in the $x-y$ plane. However, If a graph of $x-t^{2}$ is plotted, we find a straight line which has a slope of $\frac{1}{2} a$ and it will pass through the origin.

### 1.3. Experimental Procedure

### 1.3.1. Straight Line Motion with Constant Velocity

In this section of the experiment, you will study and calculate the velocity of an object moving in a straight line with a constant velocity.

1. Level the air table glass plate horizontally by using the adjustable legs.
2. Place the black carbon paper ( $50 \times 50 \mathrm{~cm}$ ) which is semiconducting on the glass plate. The carbon paper should be flat and on the air table.
3. Place white recording paper as data sheet on the flat carbon paper.
4. Place two pucks on white paper. Keep one of the pucks stationary on a folded piece of data sheet at one corner of the air table.
5. For the alignment of the air table, adjust the legs of the air table so that the puck will come to rest about the center of the table.
6. Test both two switches for the compressor and spark timer operations. With the puck pedal, the single puck should move easily, almost without friction when compressor works. When the spark timer foot switch is pressed, black dots on white paper should be observed (on the side that faces the carbon paper).
7. Set the spark timer to $f=20 \mathrm{~Hz}$.
8. Now again, test the compressor only by pressing the puck footswitch. Make sure that the puck is moving freely on the air table. By activating both the puck pedal and spark timer pedal (foot switches) in the same time, test also the spark timer and observe the black dots on the recording paper.
9. Place the puck at the edge of the table then press both compressor and spark timer pedals as you push the puck on the surface of air table. It will move along the whole diagonal distance across the air table in a straight line with constant velocity. Then, stop the pedals.
10. Remove the white recording paper from air table. The dots on the data sheet will look like those given in the Fig.1.3.


Figure 1.3. The dots produced by the puck on the data sheet.
11. Enumerate dots as $0,1,2, \ldots$. Measure the distances of the first " 5 " dots starting from dot " 0 " (as shown in Fig2.3). And then, find the time corresponding to each dot. The time between two dots is $1 / 20$ seconds since the spark timer frequency was set to $f=20 \mathrm{~Hz}$.
12. Plot the $x$-t graph. The graph must show a linear function.
13. Draw the best line that fits a linear graph. Then, calculate the velocity of the puck by using the slope of the line.

### 1.3.2. Straight Line Motion with a Constant Acceleration

In this part of the experiment, you will examine straight-line motion of an object (puck) with a constant acceleration on an inclined frictionless air table. By plotting the experimental data, you will find the acceleration of the puck sliding down on an inclined air table.

1. To perform this experiment, first place a sheet of carbon paper and then a sheet of white paper on the air table.
2. Place the foot leveler to the upper leg of the air table to give an inclination angle (the angle with the horizontal plane) as $\theta=9^{\circ}$. Use an angle finder to measure the inclination angle.
3. Put the puck at the top of the inclined plane and press the compressor pedal and check if the puck is falling freely.
4. Set the spark timer frequency to $f=20 \mathrm{~Hz}$
5. Put the puck at the top of the inclined plane to start the experiment. Press both puck and spark timer switches simultaneously and stop pressing when the puck reaches the bottom part of the inclined plane.
6. Remove the data recording paper from air table and examine the dots produced on it. Take the positive $x$-axis as direction of the puck's motion. Number and circle the dots from 0 to 5. Take the first dot as your initial data point $\left(x_{0}=0, \mathrm{t}_{0}=0\right)$, measure the distance from the other four dots to 0 dots. Determine the time of the first five dots starting from " 0 ".
7. Plot $x$ versus $t^{2}$. Then, draw the best line that fits your data points and using the slope of this line. Determine the acceleration, a, of the puck.

## 2. SIMPLE PENDULUM

Equipment : Pendulum cord, pendulum mass, photogate timer unit and connection cables Purpose : To study the motion of a simple pendulum, to learn the relationships between the period, frequency, amplitude and length of a simple pendulum, to determine the acceleration due to gravity using the motion of a simple pendulum

### 2.1. Experimental Principle

A simple pendulum consists of a small object (the pendulum bob) hung from the end of a lightweight cord. We assume that the cord does not stretch and that its mass can be ignored relative to that of the bob. The bob is free to swing back and forth through the pendulum's pivot point. The motion of a simple pendulum moving back and forth with negligible friction resembles simple harmonic motion. The pendulum oscillates along the arc with an equal amplitude on either side of its equilibrium point and as it passes through the equilibrium point with its maximum speed (Fig.2.1).

We present the forces on the mass in terms of tangential and radial components. The path of the point mass is an arc with radius $\ell$ equal to the length of the cord. The displacement of the pendulum along the arc is given by:

$$
\begin{equation*}
x=\ell \theta \tag{1}
\end{equation*}
$$

Here $\theta$ is the angle in radians that the cord makes with the vertical line and, $\ell$ is the length of the cord

The equilibrium position of the simple pendulum corresponds to the situation in which the mass is stationary and hanging vertically down $\left(\theta=0^{\circ}\right)$.

The restoring force is the net force on the bob, which is equal to the component of the weight, $m g$, tangent to the arc:

$$
\begin{equation*}
F=-m g \sin \theta \tag{2}
\end{equation*}
$$

where $g$ is the gravitational acceleration.
The minus sign here means that the force is in the opposite direction to the angular displacement, $\theta$.

If the restoring force is proportional to $x$ or $\theta$, the motion will be simple harmonic. The tangential component of the gravitational force is a restoring force that brings the pendulum back to its central (equilibrium) position. Here the restoring force is proportional to $\sin \theta$ instead of $\theta$, so the motion is not simple harmonic. However, if the angle $\theta$ is infinitesimal, $\sin \theta$ is approximately equal to $\theta$ in radians. For example, when $\theta=0.1 \mathrm{rad}$ (about $\left.60^{\circ}\right), \sin \theta=0.0998$, a difference of only $0.2 \%$.
(a)


Equilibrium position
(b)


Figure 2.1. Simple pendulum consists of a small bob of mass suspended by a massless cord of length (a). The restoring force is the tangential component of the net force (b)

Note that the arc length $x(=\boldsymbol{\ell \theta})$ is nearly equal to the straight line $(=\ell \sin \theta)$ as seen in the Fig.3.1b if $\theta$ is small. For angles less than $15^{\circ}$, the difference between $\theta$ and $\sin \theta$ is less than $1 \%$.

So, the maximum angle of swing must be small. For small angles, $\sin \theta$ can be approximated by $\theta$ (in radians) so that Eq. 2 can be written as:

$$
\begin{align*}
& F=-m g \theta  \tag{3}\\
& F=-m g \frac{x}{\ell}  \tag{4}\\
& F=-\frac{m g}{\ell} x \tag{5}
\end{align*}
$$

For small displacements, the motion is essentially simple harmonic, since this equation fits Hooke's law, $F=-k x$.

If the motion is simple harmonic, as we have seen, the restoring force must be directly proportional to $x$ (since $x=\ell \theta$ ). The restoring force is then proportional to the coordinate for small displacements and the force constant, $k$ is given by:

$$
\begin{equation*}
k=\frac{m g}{\ell} \tag{6}
\end{equation*}
$$

The angular frequency, $\omega$ of a simple pendulum with small amplitude is given by:

$$
\begin{equation*}
\omega=\sqrt{\frac{k}{m}} \tag{7}
\end{equation*}
$$

By using Eq.6, we get:

$$
\begin{equation*}
\omega=\sqrt{\frac{m g / \ell}{m}} \tag{8}
\end{equation*}
$$

So, we conclude that the angular frequency of small oscillations of a simple pendulum is given by:

$$
\begin{equation*}
\omega=\sqrt{\frac{g}{\ell}} \tag{9}
\end{equation*}
$$

In this case, the pendulum's frequency is dependent only on the length of the pendulum and the acceleration due to gravity. However, it is independent of the mass of the pendulum and the amplitude of the pendulum swings (provided that $\sin \theta \approx \theta$ ). The corresponding frequency, $f$ and period, $T$ relationships will be:

$$
\begin{gather*}
f=\frac{\omega}{2 \pi}  \tag{10}\\
T=\frac{1}{f}=\frac{2 \pi}{\omega}  \tag{11}\\
T=2 \pi \sqrt{\frac{\ell}{g}} \tag{12}
\end{gather*}
$$

When the angular displacement is $\theta<15^{\circ}$, the period of a pendulum can be determined with the Eq. 12 . For small amplitudes, the period of a simple pendulum depends only on its length and the value of the gravitational acceleration.

By measuring the length and period of the pendulum, we can determine the gravitational acceleration $(g)$ by the Eq. 12:

$$
\begin{equation*}
T^{2}=4 \pi^{2} \frac{\ell}{g} \tag{13}
\end{equation*}
$$

$$
\begin{equation*}
g=4 \pi^{2} \frac{\ell}{T^{2}} \tag{14}
\end{equation*}
$$

As seen from the Eq.14, the gravitational acceleration can be determined experimentally for different lengths of the pendulum by means of the oscillating period.

### 2.2. Experimental Procedure

1. Construct the simple pendulum apparatus.
2. Attach the mass (bob) to the cord of the pendulum so that the pendulum bob hangs vertically downward approximately 50 cm from the pivot point.
3. Start the photogate timer unit.
4. Pull the mass back such that the rod makes an angle of $\theta=5^{\circ}$ with its equilibrium position.
5. Measure the period $(T)$ of the pendulum using Number- 3 button on the photogate timer unit when it is displaced $\theta=5^{\circ}$ from its equilibrium position.
6. Calculate the gravitational acceleration $(g)$ using the period of the pendulum. Record the values in the appropriate column of your data table.
7. Compare your experimental value of the gravitational acceleration with the accepted value of $g=9.80 \mathrm{~m} / \mathrm{s}^{2}$ to obtain percentage error.
8. Now, you will calculate the angular frequency ( $\omega$ ) of the pendulum using its length and the acceleration $(g)$ that you have found experimentally and the frequency $(f)$ of the simple pendulum from Eq. 10 .
9. Repeat all the experimental procedures made in the previous steps for different lengths of the pendulum while keeping the mass and angle constant $\left(\boldsymbol{\theta}=\mathbf{5}^{\circ}\right)$ :

- $l=60 \mathrm{~cm}$.
- $\quad l=65 \mathrm{~cm}$.

10. Now, you will determine experimentally the acceleration of gravity, angular frequency and frequency for different displacement angle $\left(\theta=10^{\circ}\right)$.
11. Record all values in the appropriate columns of your data tables-1, 2, 3, 4 .

Table 2.1.

| $\theta$ (degree) | $m$ (kg) | $L(m)$ | $T$ (s) | $\begin{gathered} g\left(\mathrm{~m} / \mathrm{s}^{2}\right) \\ \text { Experimental } \end{gathered}$ | $g\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ <br> Accepted | Percentage Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 |  | 0.50 |  |  |  |  |
|  |  | 0.60 |  |  |  |  |
|  |  | 0.65 |  |  |  |  |

Table 2.2.

| $\boldsymbol{\theta}($ degree $)$ | $\boldsymbol{m}(\mathrm{kg})$ | $L(\mathrm{~m})$ | $\boldsymbol{T}(\mathbf{s})$ | $\omega\left(\mathbf{s}^{-1}\right)$ <br> Experimental | $\omega\left(\mathbf{s}^{-1}\right)$ <br> Accepted | Percentage <br> Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 |  |  | 0.50 |  |  |  |

Table 2.3.

| $\theta(d e g r e e)$ | $m(k g)$ | $L(m)$ | $T(s)$ | $g\left(m / s^{2}\right)$ <br> Experimental | $g\left(\mathbf{m} / \mathbf{s}^{2}\right)$ <br> Accepted | Percentage <br> Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10 | 0.50 |  |  |  |  |
|  |  | 0.60 |  |  |  |  |
|  |  | 0.65 |  |  |  |  |

Table 2.4.

| $\boldsymbol{\theta}$ (degree) | $\boldsymbol{m}(\mathrm{kg})$ | $L(\mathrm{~m})$ | $\boldsymbol{T}(\mathrm{s})$ | $\omega\left(\mathbf{s}^{-1}\right)$ <br> Experimental | $\omega\left(\mathbf{s}^{-1}\right)$ <br> Accepted | Percentage <br> Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 |  |  | 0.50 |  |  |  |
|  |  | 0.60 |  |  |  |  |
|  |  | 0.65 |  |  |  |  |

## 3. ANALYSIS OF A SPRING

Equipment : Two types of helical springs, various masses, stopwatch, ruler
Purpose : Determination of spring constants of helical springs by Hooke's Law and vibration method

### 3.1. Experimental Principle

### 3.1.1 Hooke's Law

Real materials are not perfectly rigid. When subjected to forces, they can deform. If a body deforms when subjected to a force, it returns to its initial shape when the force is removed, the body is elastic. Main reason to get back to the original shape is internal restoring forces. A helical spring is a very simple example of an elastic body (see in Fig.3.1). If deviation $\Delta l$, ( $\left.\Delta l=\left|x_{1}-x_{0}\right|\right)$ from the equilibrium position $l_{0}$ of the helical spring is not very large, the restoring force $F$ of the spring is found to be proportional to its elongation $\Delta l$ :

$$
\begin{equation*}
F=-k . \Delta l \tag{1}
\end{equation*}
$$

This is Hooke's law, where the proportionality constant $k$, which is a general magnitude of reference, is called as spring constant of the helical spring. If an external force acts on the spring, such as the weight $F=m . g$ of a mass $m\left(g=9.81 \mathrm{~m} / \mathrm{s}^{2}:\right.$ gravitational acceleration $)$, for which the weight of the mass $m$ is equal to the restoring force of the spring:

$$
\begin{equation*}
F=k . \Delta l=m . g \tag{2}
\end{equation*}
$$



Figure 3.1. Measurement of the elongation of the helical spring

### 3.1.2 Springs Connected in Series

If an external force $F$ acts on a spring system which is connected in series, force felt by each spring would be the same. If the force acting on the spring system is $F$ and forces experienced by each spring are $F_{1}$ and $F_{2}$, we can write;

$$
\begin{equation*}
F=F_{1}=F_{2} \tag{3}
\end{equation*}
$$

For the elongations, the relationship can be written:

$$
\begin{equation*}
\Delta l=\Delta l_{1}+\Delta l_{2} \tag{4}
\end{equation*}
$$

If we substitute Eq. 3 and Eq. 4 in Eq.1, we get the spring constant as

$$
\begin{align*}
& \frac{F}{k}=\frac{F}{k_{1}}+\frac{F}{k_{2}}  \tag{5}\\
& \frac{1}{k}=\frac{1}{k_{1}}+\frac{1}{k_{2}} \tag{6}
\end{align*}
$$

or

$$
\begin{equation*}
k=\frac{k_{1} \cdot k_{2}}{k_{1}+k_{2}} \tag{7}
\end{equation*}
$$

### 3.1.3 Springs Connected in Parallel

If the ends of two springs with spring constants $k_{1}$ and $k_{2}$ are connected side-by-side, the type of the connection is called parallel. When an external force $F$ acts on the system, the elongations of each spring are the same and the total force is sum of the forces experienced by each spring:

$$
\begin{gather*}
F=F_{1}+F_{2}  \tag{8}\\
\Delta l=\Delta l_{1}=\Delta l_{2} \tag{9}
\end{gather*}
$$

Therefore, sum of the forces acting on each springs in the spring system equal to force acting on the system, and also the elongations or compressions of the springs equal to each other. Using these relations, spring constant of the system can be written as

$$
\begin{gather*}
k \cdot x=k \cdot x_{1}+k \cdot x_{2} \\
k=k_{1}+k_{2} \tag{10}
\end{gather*}
$$

As seen in Eq.10, when the springs are connected in parallel, the equivalent spring constant of the system would be greater than the spring constants of each spring.

### 3.1.4 Vibration Method

While an body of mass $m$ hung the end of the spring is in the equilibrium position $\Delta l=0$, if it is pulled down and released, restoring force of the spring causes acceleration toward the equilibrium position and thus mass starts to do simple harmonic motion. When the body is oscillating, the time for one complete oscillation is called period, $T$. Force acting on the mass, $m$ during simple harmonic motion is equal to $F=-k . \Delta l$. The magnitude of the force, $F$, changes with the $\Delta l$. Therefore, force acting on the body is proportional to the distance between the body and the equilibrium position. In the simple harmonic motion, another parameter is frequency $f$, which is defined as the number of oscillation per unit time. The relation between frequency and period is

$$
\begin{equation*}
T=\frac{1}{f} \tag{11}
\end{equation*}
$$

The period $T$ is determined by the spring constant $k$ and the mass $m$ hung the spring:

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{m}{k}} \tag{12}
\end{equation*}
$$

If Eq. 12 is modified, the spring constant is obtained as

$$
\begin{equation*}
k=\frac{4 \pi^{2} m}{T^{2}} \tag{13}
\end{equation*}
$$

### 3.2. Experimental Procedure

### 3.2.1 Determination of spring constants using Hooke's Law

1. Prepare experimental setup as in Fig.3.2.
2. First, attach the pan to end of thin spring and determine the place of the equilibrium position using the ruler.
3. Put various masses $m$ into pan, measure the elongations $(\Delta l)$ from the equilibrium position for each mass and write the values into Table 3.1.
4. Draw a graph showing mass $m$ on vertical axis and corresponding elongations ( $\Delta l$ ) on horizontal axis.
5. Calculate $k_{\text {thin }}$ using slope of graph.
6. Repeat the same procedures from 1 to 5 for thick spring and calculate $k_{\text {thick }}$.
7. Connect to the springs in series and apply the same procedures and calculate $k_{\text {serial }}$. Calculate $k_{\text {serial }}$ using Eq. 7 and compare this experimental $k_{\text {serial }}$.
8. Calculate the error.


Figure 3.2. Hooke's Law Experimental Set-up
Table 3.1: Data Table for Hooke's Law Experiment

| Thin spring |  | Thick spring |  | Springs connected in serial |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{m}(\boldsymbol{g})$ | $\Delta \boldsymbol{l}(\boldsymbol{c m})$ | $\boldsymbol{m}(\boldsymbol{g})$ | $\Delta \boldsymbol{l}(\boldsymbol{c m})$ | $\boldsymbol{m}(\boldsymbol{g})$ | $\Delta \boldsymbol{l}(\boldsymbol{c m})$ |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

### 3.2.2 Determination of spring constants using vibration method

1. Put a mass of 250 g into pan when thin spring is in hanger.
2. Up slightly lift the mass from the equilibrium position and then release it. See vibrational motion.
3. Measure time spent for 10 oscillations using stopwatch when the spring and mass are doing a vibrational motion.
4. Calculate $T_{\text {thin }}$ using data, which was found before.
5. Calculate $k_{\text {thin }}$ using Eq. 13 .
6. Repeat the same procedures for thick spring and calculate $k_{\text {thick }}$.
7. Calculate the error.

Note: To calculate $m$ value, which is needed for Eq. 13 , use equation $m=m_{\text {pan }}+m_{\text {masses }}+\frac{m_{\text {spring }}}{3}$ (Why?)

## 4. CONSERVATION OF ENERGY

$\begin{array}{lll}\text { Equipment } & \text { : } & \text { Pulley, } 2 \text { optic gates, mass hanger and mass set, air rail , air pump } \\ \text { Purpose } & : & \text { Studying law of conservation of energy }\end{array}$

### 4.1. Experimental Principle

In mechanics it is natural that we should be concerned primarily with the various mechanical forms of energy. It is convenient to distinguish between two types - potential energy and kinetic energy.

The energy that a body has depending on its position or configuration is called potential energy. The measure of the potential energy, $U$ is the work done against gravity in lifting the body. The upward force required is equal to the weight of body, and the work done in lifting the body through a height $h$ is given by;

$$
\begin{equation*}
U=m g h \tag{1}
\end{equation*}
$$

Kinetic energy is the energy a body possesses depending on its motion. To find the kinetic energy, which a body has, we consider the work, which must be done on the body in order to give it its speed. The work $W$ done on the body of mass $m$, which initially at rest is

$$
\begin{equation*}
W=F x \tag{2}
\end{equation*}
$$

where $F$ is the force applied to the body and $x$ is the displacement. By Newton's second law $F=m a$; thus the work is equal to max. Since the acceleration of the body is constant and $v_{0}=0$,

$$
\begin{equation*}
v^{2}=2 a x \tag{3}
\end{equation*}
$$

If we substitute Eq. 3 with Eq. 2 and recall that the work done appears as kinetic energy, we have

$$
\begin{equation*}
K=\frac{1}{2} m v^{2} \tag{4}
\end{equation*}
$$

Much of physics involves the relationships among the many forms of energy and the transformations from one to another. The study of these transformations has led to the statement of a very important principle, known as the law of conservation of energy: Energy cannot be created or destroyed; it may be transformed from one form to another, but the total amount of energy never changes.

According to the law of conservation of energy, if there are only conservative forces (no frictional force or no external force but the gravitation) acting on a body, the mechanical energy never changes. We can describe the mechanical energy, $E$ as sum of the kinetic and the potential energies:

$$
\begin{equation*}
E=K+U \tag{5}
\end{equation*}
$$

### 4.2. Experimental Procedure

1. Attach some masses to mass hanger and record the total mass as $m$
2. Place the sled of mass $M$ just front of the first optic gate
3. Measure the height of the $m$ from the ground and record as $h_{1}$
4. Calculate total initial energy $E_{i}$
5. Start up the timer which measures transition time of the sled between two optic gates
6. Turn the air pump on. When the pump begins to operate, the system will move together.
7. When the sled exits from the second optic gate, measure the height of $m$ again and record as $h_{2}$
8. Calculate the total final energy $E_{f}$
9. Compare your results and discuss
10. Calculate the error.


Figure 4.1. Experimental setup

